Applied Mathematics and Computational Science Qualifying Exam Guidelines

Department of Mathematics and Computer Science, Chulalongkorn University

SUBJECT: Applied Linear Algebra

1. **Systems of linear equations**

   - Gaussian elimination, triangular factorizations
   - Theory of simultaneous linear equations

2. **Vector spaces and linear transformations**

   - Vector spaces, subspaces, basis, dimension, coordinates
   - Fundamental subspaces, fundamental theorems
   - Linear transformations, matrix representation

3. **Orthogonal projections and least squares**

   - Inner products, orthogonality
   - Projections and least square approximations
   - Orthogonal bases, Gram-Schmidt process, QR factorization

4. **Eigenvalue-eigenvector**

   - Determinants and applications
   - Eigenvalues, eigenvectors
   - Similar transformation and diagonalization
   - Applications in difference and differential equations
   - Hermitian, unitary, Schur's lemma
   - Jordan form, singular values, SVD
   - Estimation of eigenvalues

5. **Positive definite**

   - Positive definite (semi-definite) matrices and testing methods
   - Min/max principles and Rayleigh's quotient
   - Rayleigh-Ritz principle and application to finite element method

6. **Computation with matrices**

   - Norms, condition numbers and relative errors for Ax=b.
   - Iterative methods for Ax=b (Jacobi, Gauss-Seidel, SOR)